Name_____

Books and notes are closed. Calculators are open, loaded with any information you please; calculators may not be lent or borrowed. Intermediate work must be shown.

Hand this question sheet in with your answer booklets, and make sure each booklet has your name.

1 Solve the inequality $|2x - 13| < 151$.

2a Define the convergence of the sequence a_n to the limit L. Be rigorous.
2b Prove, using ϵ notation, that the sequence $a_n = \frac{1}{n\sqrt{n}}$ converges, and find the limit.
2c Use the squeeze theorem (sandwich theorem) to find the limit of $\{\frac{\sin n}{n\sqrt{n}}\}$, justifying your answer.

3 Set up the integral to find the area in the first quadrant between the curves $y = x^3$ and $y = \sqrt[5]{x}$. Then solve.

4a Give the definition of "derivative" using limits. Use that definition to find the velocity if the position is $\frac{5}{1+t}$. No quick Newton formula, please.
4b Take the derivative with respect to x of $\cos(\frac{\cos x}{x})$, using Newton's formulas and methods.

5a If $y^4 + x^4 + x^2y = 3$, find an equation of the tangent to that curve at (1,1).
5b If displacement is given by $s(t) = 9 + \frac{1}{5}\cos(11\pi t)$, where t is time, find velocity and acceleration.

6 Graph the curve $y = x^2 - x^3$ on [0,1]. Show and label the intervals where it is increasing, decreasing, concave upward, or concave downward; show and label any inflection points. Find the absolute maximum and absolute minimum on [0,1].

7 Prove that the equation $1 + 3x + x^3 + 7x^5 = 0$ has exactly one root.

8 The radius of a right circular cylinder is increasing at six feet per minute. The volume is increasing at two cubic feet per minute. How fast is the height increasing when the radius is three feet and the height is one foot?

9 Get an approximation to $\sqrt[3]{27.1}$ using differentials (or, equivalently, using a linear approximation.)

10 Use calculus to find the dimensions of a rectangle with perimeter 500 feet whose area is as small as possible.

11a Evaluate $\displaystyle\int_2^3 x^{-6} - 16x + \frac{1}{\sqrt{x}}\,dx.$

11b Evaluate $\displaystyle\int_4^9 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx.$

11c Evaluate $\displaystyle\int_0^{\pi/2} \frac{\cos x}{(2+\sin x)^2}\,dx.$

12 Use calculus to find the point on the line $y = x^2$ which is closest to the point (3,1).

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